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A Perspective on the WPL Theory

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ARTICLE

A Perspective on Thirty Years of the Webb, Pearman and Leuning Density Corrections

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Abstract The density correction theory of Webb et al. (1980, Q J Roy Meteorol Soc 106: 85–100, hereafter WPL) is a principle underpinning the experimental investigation of surface fluxes of energy and masses in the atmospheric boundary layer. It has a long-lasting influence in boundary-layer meteorology and micrometeorology, and the year 2010 marks the 30th anniversary of the publication of the WPL theory. We provide here a critique of the theory and review the research it has spurred over the last 30 years. In the authors' opinion, the assumption of zero air source at the surface is a fundamental novelty that gives the WPL theory its enduring vitality. Considerations of mass conservation show that, in a non-steady state, the WPL mean vertical velocity and the thermal expansion velocity are two distinctly different quantities of the flow. Furthermore, the integrated flux will suffer a systematic bias if the expansion velocity is omitted or if the storage term is computed from time changes in the CO_2 density. A discussion is provided on recent efforts to address several important practical issues omitted by the original theory, including pressure correction, unintentional alternation of the sampled air, and error propagation. These refinement efforts are motivated by the need for an unbiased assessment of the annual carbon budget in terrestrial ecosystems in the global eddy flux network (FluxNet).

WPL in steady state

$$\mathrm{NEE} = \overline{w}\,\overline{\varrho}_c + \overline{w'\varrho_c'}$$

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$$\mathrm{NEE} = \overline{w}\,\overline{\varrho}_c + \overline{w'\varrho_c'}$$

$$= [-\frac{1}{\varrho_d}\overline{w'\varrho_d'}]\,\overline{\varrho}_c + \overline{w'\varrho_c'}$$

WPL velocity

Inferences from the WPL theory

- No correction is required if the mixing ratio is used instead of the density in eddy-covariance flux calculation
- Online point-by-point conversion to mixing ratio should yield the same result as the off-line correction based on (half-hourly) statistics
- No temperature correction is required if temperature fluctuations are removed prior to the measurement of the CO_2 density
- No water vapor density correction is required if water vapor is removed prior to the measurement of the CO_2 density
- Artificial density fluctuations caused by sensor self-heating cannot be corrected with the standard heat flux measurement
- It is incorrect to force energy balance closure on sensible and latent heat fluxes before implementing the WPL correction

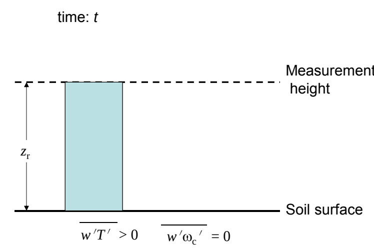
WPL in non-steady state

$$\mathrm{NEE} = \int_0^z \frac{\partial \overline{\varrho_c}}{\partial t}\, dz' + \Big[-\frac{1}{\overline{\varrho_d}} \int_0^z \frac{\partial \overline{\varrho_d}}{\partial t}\, dz'\Big]\, \overline{\varrho}_c + \Big[-\frac{1}{\overline{\varrho_d}} \overline{w'\varrho_d'}\Big]\, \overline{\varrho}_c + \overline{w'\varrho_c'}$$

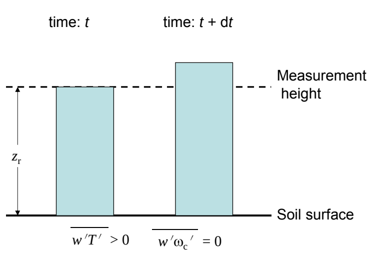
Storage — Column expansion velocity — WPL velocity

$$\overline{w}_e = -\frac{1}{\overline{\varrho}_d} \int_0^z \frac{\partial \overline{\varrho_d}}{\partial t} dz'$$

Column expansion velocity

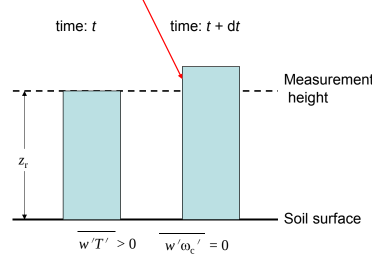


Column expansion velocity



Column expansion velocity

$$\overline{w}_e = -\frac{1}{\overline{\varrho}_d} \int_0^z \frac{\partial \overline{\varrho_d}}{\partial t} dz'$$



WPL in non-steady state

$$\mathrm{NEE} = \int_0^z \frac{\partial \overline{\varrho_c}}{\partial t}\, dz' + \Big[-\frac{1}{\overline{\varrho_d}} \int_0^z \frac{\partial \overline{\varrho_d}}{\partial t}\, dz'\Big]\, \overline{\varrho}_c + \Big[-\frac{1}{\overline{\varrho_d}} \overline{w'\varrho_d'}\Big]\, \overline{\varrho}_c + \overline{w'\varrho_c'}$$

Quasi-advection flux

WPL in non-steady state

$$\mathrm{NEE} = \int_0^z \frac{\partial \overline{\varrho_c}}{\partial t}\, dz' + \Big[-\frac{1}{\overline{\varrho_d}} \int_0^z \frac{\partial \overline{\varrho_d}}{\partial t}\, dz'\Big]\, \overline{\varrho}_c + \Big[-\frac{1}{\overline{\varrho_d}} \overline{w'\varrho_d'}\Big]\, \overline{\varrho}_c + \overline{w'\varrho_c'}$$

$$= \int_0^z \frac{\partial \overline{\varrho_c}}{\partial t}\, dz' + \Big[-\frac{1}{\overline{\varrho_d}} \int_0^z \frac{\partial \overline{\varrho_d}}{\partial t}\, dz'\Big]\, \overline{\varrho}_c + \overline{\varrho}_d\, \overline{w'\chi_c'}$$

WPL in non-steady state

$$\mathrm{NEE} = \int_0^z \overline{\frac{\partial \varrho_c}{\partial t}}\, dz' + [-\frac{1}{\varrho_d}\int_0^z \overline{\frac{\partial \varrho_d}{\partial t}}\, dz']\,\overline{\varrho}_c + [-\frac{1}{\varrho_d}\overline{w'\varrho_d'}]\,\overline{\varrho}_c + \overline{w'\varrho_c'}$$

$$= \int_0^z \overline{\frac{\partial \varrho_c}{\partial t}}\, dz' + [-\frac{1}{\varrho_d}\int_0^z \overline{\frac{\partial \varrho_d}{\partial t}}\, dz']\,\overline{\varrho}_c + \overline{\varrho}_d\,\overline{w'\chi_c'}$$

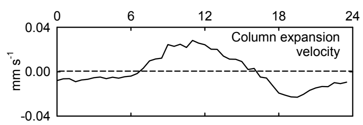
$$= \overline{\varrho}_d \int_0^z \overline{\frac{\partial \chi_c}{\partial t}}\, dz' + \overline{\varrho}_d\,\overline{w'\chi_c'}$$

WPL in non-steady state

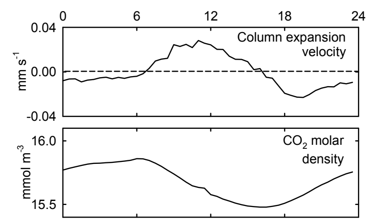
$$\mathrm{NEE} = \int_0^z \overline{\frac{\partial \varrho_c}{\partial t}}\, dz' + [-\frac{1}{\varrho_d}\int_0^z \overline{\frac{\partial \varrho_d}{\partial t}}\, dz']\,\overline{\varrho}_c + [-\frac{1}{\varrho_d}\overline{w'\varrho_d'}]\,\overline{\varrho}_c + \overline{w'\varrho_c'}$$

Quasi-advection flux

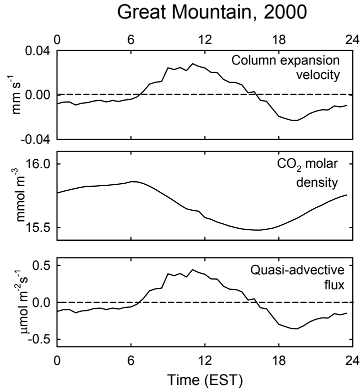
Quasi-advective CO_2 flux
Great Mountain, 2000



Quasi-advective CO_2 flux
Great Mountain, 2000



Quasi-advective CO_2 flux
Great Mountain, 2000



Total mean vertical velocity

$$W = -\frac{1}{\overline{\varrho}_d}\overline{w'\varrho_d'}$$

WPL velocity

Total mean vertical velocity

$$W = -\frac{1}{\overline{\varrho}_d}\overline{w'\varrho_d'}$$ WPL velocity

$$-\frac{1}{\overline{\varrho}_d}\int_0^z \frac{\partial \overline{\varrho}_d}{\partial t}\,dz'$$ Column expansion velocity

Total mean vertical velocity

$$W = -\frac{1}{\overline{\varrho}_d}\overline{w'\varrho_d'}$$ WPL velocity

$$-\frac{1}{\overline{\varrho}_d}\int_0^z \frac{\partial \overline{\varrho}_d}{\partial t}\,dz'$$ Column expansion velocity

$$-\frac{1}{\overline{\varrho}_d}\int_0^z \boldsymbol{\nabla_H} \bullet (\overline{\mathrm{u}}\,\overline{\varrho}_d)\,dz'$$ Velocity caused by flow convergence

WPL in non-steady state and advective flow

$$\mathrm{NEE} \neq W\,\overline{\varrho}_c + \overline{w'\varrho_c'}$$

WPL in non-steady state and advective flow

$$\mathrm{NEE} = \int_0^z \frac{\partial \overline{\varrho}_c}{\partial t}\,dz'$$

Storage

WPL in non-steady state and advective flow

$$\mathrm{NEE} = \int_0^z \frac{\partial \overline{\varrho}_c}{\partial t}\,dz' + \left[-\frac{1}{\overline{\varrho}_d}\int_0^z \frac{\partial \overline{\varrho}_d}{\partial t}\,dz'\right]\overline{\varrho}_c$$

Quasi-advection flux

WPL in non-steady state and advective flow

$$\mathrm{NEE} = \int_0^z \frac{\partial \overline{\varrho}_c}{\partial t}\,dz' + \left[-\frac{1}{\overline{\varrho}_d}\int_0^z \frac{\partial \overline{\varrho}_d}{\partial t}\,dz'\right]\overline{\varrho}_c + \left[-\frac{1}{\overline{\varrho}_d}\overline{w'\varrho_d'}\right]\overline{\varrho}_c$$

WPL term

WPL in non-steady state and advective flow

$$\mathrm{NEE} = \int_0^z \overline{\frac{\partial \varrho_c}{\partial t}}\, dz' + [-\frac{1}{\varrho_d}\int_0^z \overline{\frac{\partial \varrho_d}{\partial t}}\, dz']\,\overline{\varrho}_c + [-\frac{1}{\varrho_d}\overline{w'\varrho_d'}]\,\overline{\varrho}_c + \overline{w'\varrho_c'}$$

Eddy flux

WPL in non-steady state and advective flow

$$\mathrm{NEE} = \int_0^z \overline{\frac{\partial \varrho_c}{\partial t}}\, dz' + [-\frac{1}{\varrho_d}\int_0^z \overline{\frac{\partial \varrho_d}{\partial t}}\, dz']\,\overline{\varrho}_c + [-\frac{1}{\varrho_d}\overline{w'\varrho_d'}]\,\overline{\varrho}_c + \overline{w'\varrho_c'} + \overline{w}_a\,[\overline{\varrho}_c(z) - \frac{1}{z}\int_0^z \overline{\varrho}_c\, \mathrm{d}z']$$

Vertical advection

WPL in non-steady state and advective flow

$$\mathrm{NEE} = \int_0^z \overline{\frac{\partial \varrho_c}{\partial t}}\, dz' + [-\frac{1}{\varrho_d}\int_0^z \overline{\frac{\partial \varrho_d}{\partial t}}\, dz']\,\overline{\varrho}_c + [-\frac{1}{\varrho_d}\overline{w'\varrho_d'}]\,\overline{\varrho}_c + \overline{w'\varrho_c'} + \overline{w}_a\,[\overline{\varrho}_c(z) - \frac{1}{z}\int_0^z \overline{\varrho}_c\, \mathrm{d}z']$$

WPL in non-steady state and advective flow

$$\mathrm{NEE} = \int_0^z \overline{\frac{\partial \varrho_c}{\partial t}}\, dz' + [-\frac{1}{\varrho_d}\int_0^z \overline{\frac{\partial \varrho_d}{\partial t}}\, dz']\,\overline{\varrho}_c + [-\frac{1}{\varrho_d}\overline{w'\varrho_d'}]\,\overline{\varrho}_c + \overline{w'\varrho_c'} + \overline{w}_a\,[\overline{\varrho}_c(z) - \frac{1}{z}\int_0^z \overline{\varrho}_c\, \mathrm{d}z']$$

$$= \overline{\varrho_d}\int_0^z \overline{\frac{\partial \chi_c}{\partial t}}\, dz' + \overline{\varrho}_d\,\overline{w'\chi_c'} + \overline{w}_a\,[\overline{\varrho}_c(z) - \frac{1}{z}\int_0^z \overline{\varrho}_c\, \mathrm{d}z']$$

Refined WPL

$$F_{d,T} = (1 + \mu_v \overline{\omega}_v)\frac{\overline{w'T'}}{\overline{T}}\,\overline{\rho}_c,$$

$$F_{d,v} = \mu_v \frac{\overline{w'\rho_v'}}{\overline{\rho}_d}\,\overline{\rho}_c,$$

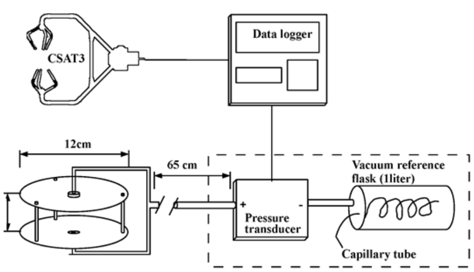
$$F_{d,p} = -(1 + \mu_v \overline{\omega}_v)\frac{\overline{w'p'}}{\overline{p}}\,\overline{\rho}_c,$$

$$F_{d,c} = \mu_c \frac{\overline{w'\rho_c'}}{\overline{\rho}_d}\,\overline{\rho}_c,$$

$$F_{d,h} = -(1 + \mu_v \overline{\omega}_v)\frac{1}{\overline{T}}(\frac{\overline{w'T'^2}}{T})\,\overline{\rho}_c.$$

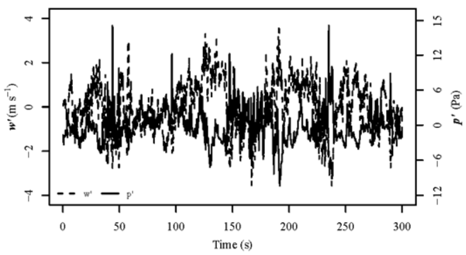
Source: Fuehrer and Friehe (2004) *Bound-Layer Met* 102:415-457

Measuring pressure flux
CBS Forest, Northeast China



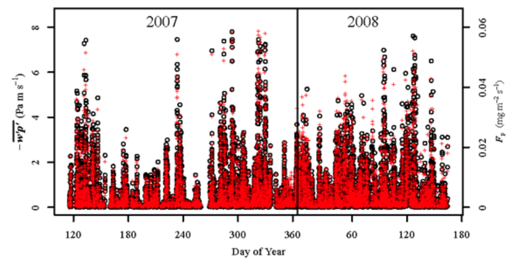
Source: Zhang et al. (2011) *Agric For Met* 151:70-77

Pressure and vertical velocity fluctuations
CBS Forest, Northeast China



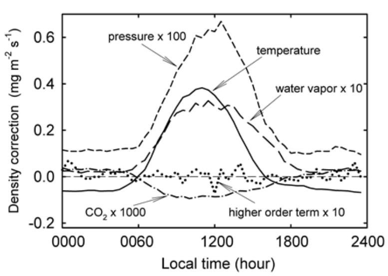
Source: Zhang et al. (2011) *Agric For Met* 151:70-77

Pressure flux and the associated density correction
CBS Forest, Northeast China



Source: Zhang et al. (2011) *Agric For Met* 151:70-77

Diurnal Composites of the Density Correction
Mixed Forest, Northeast China



Source: Lee and Massman (2011) *Bound-Layer Met* 139:37-59

Annual mean correction (tC ha^{-1} y^{-1})
CBS Forest, Northeast China

Temperature	5.52
Water vapor	0.88
Pressure	0.40
CO_2 self-dilution	-0.05
Higher-order term	-0.001

Source: Lee and Massman (2011) *Bound-Layer Met* 139:37-59

- Quasi-advection term appears negligible for the annual carbon flux
- Pressure correction is not negligible for open-path eddy-covariance systems
- Not all vertical velocities are alike
- “Nothing in biology makes sense except in light of evolution” – Michael Donoghue

- Quasi-advection term appears negligible for the annual carbon flux
- Pressure correction is not negligible for open-path eddy-covariance systems
- Not all vertical velocities are alike
- Nothing in WPL makes sense except in light of mass conservation