The Dependence of Sea Surface Roughness on the Height and Steepness of the Waves

Peter K. Taylor and Margaret J. Yelland
Southampton Oceanography Centre, Southampton, United Kingdom

Reporter: Cao Zhengda
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Introduction

- The variation of the roughness length $z_o$ with wind speed for rough flow over the sea is first described by Charnock (1955)

  \[ z_o = z_{Ch} u_*^2 / g \]  

  where $u_*$ is the friction velocity, $g$ the acceleration due to gravity, and $z_{Ch}$ the Charnock parameter used to characterize the variation of the drag coefficient $C_{D10n}$ with wind speed $U_{10n}$ at 10m.

- Hsu (1974) suggested that $z_{Ch}$ was a function of the wave slope $(H_s / L_p)$ where $H_s$ is the significant wave height and $L_p$ the wavelength of the waves at peak of the wave spectrum.

- Using a subset of the HEXMAX data chosen to avoid the effects of swell, Matt et al. (1991) and Smith et al. (1992) proposed the “HEXOS” relationship

  \[ z_{Ch} = \alpha (c_p / u_*)^{-\beta} \]  

  where $c_p / u_*$ is defined as the wave age, and $c_p$ the phase speed at the peak of the wave spectrum. Eq. (2) implies younger waves are rougher than older waves.
Using data from Lake Ontario, Donelan (1990) developed a wave-age-based formula

$$\frac{z_o}{\eta} = 5.5 \times 10^{-4} \left( \frac{U_{10n}}{c_p} \right)^{2.7}$$

Anctil and Donelan (1996) found the slope-based formula fitted the data equally well:

$$\frac{z_o}{\eta} = 2.55 \times 10^{3} \theta^{6.76}$$

where $\theta$ is the rms slope.

And that the best fit was obtained using a combination of wave age and slope:

$$\frac{z_o}{\eta} = 2.26 \left( \frac{U_{10n}}{c_p} \right)^{1.82} \theta^{3.83}$$
Development of waves
Relationship between $C_{D10n}$ and $z_o$

\[ U_{10n} = \frac{u_*}{\kappa} \ln \frac{10}{z_o} \]

\[ C_{D10n} = \left( \frac{u_*}{U_{10n}} \right)^2 \]

\[ C_{D10n} = \kappa^2 / [\ln(10 / z_o)]^2 \]
Objective

By using a modified form of Eq. (4) resolve several major discrepancies caused by seeking relationships between the roughness and the wave age.
The datasets

- **HEXMAX**
  Fetch: $\geq 175$ km
  No swell
  For a given wind speed the longer wavelength waves were older and would have been expected to be smoother—the opposite of the observation.
The datasets

- **RASEX**
  Fetch limited:
  2-5 km for offshore wind flow
  15-25 km for onshore wind flow

  Higher $C_{D10n}$ values were observed in offshore rather than onshore flow.

  The RASEX $C_{D10n}$ values for onshore winds were much lower than would be predicted by Eq. (2).

  —Vickers and Mahrt (1997a, hereafter VM97)
The datasets

- Anctil and Donelan (1996)-Lake Ontario
  - Depth: 2, 4, 8, 12 m
  - For run 166: wind speed: 7 m/s    fetch: 8 km
  - For run 185: wind speed: 14 m/s    fetch: 300 km

  The results from this study were chosen for they were published with consistent wave information.
  The purpose of this experiment was to study the effect of shoaling waves.
Development of a roughness length formula

- Choice of scaling variables
  
  Using $H$ to scale the roughness appears a better option.

  Only by using $(H_s / L_p)$ as a scaling parameter is the higher wind speed RASEX collapsed onto the HEXOS points. Therefore adopt this scaling:

  \[ z_o / H_s = A_1 (H_s / L_p)^{B_1} \]  

  Noting that with this scaling the lower wind speed data from HEXOS and AD96 diverge, a multiple scaling similar to Eq. (5) is considered:

  \[ z_o / H_s = A_2 (H_s / L_p)^{B_2} \left( \frac{U_{10n}}{c_p} \right)^{C} \]
Development of a roughness length formula

- **Choice of coefficients**

  Using the wave age and Eq. (7) can explain little more of the variance than Eq. (6) \((R^2 = 0.58\) compared to 0.56\), and maybe introduce more chance of self-correlation, then Eq. (6) could be better.

  Obtain \(\ln(A_1) = 7.09, B_1 = 4.5\) for a two-way regression, which are more similar to the one-way regression, presumably because the scatter is dominated by the error in determining \(Z_0\).
Development of a roughness length formula

- Predicted effects of varying duration, fetch, and water depth

Eq. (6) predicts a slight decrease of compared to the value for a fully developed sea, normally by less than 10%.

At about \( h_s = 0.2L_p \) the effect of shoaling occurs.

Both Eq. (6) and the Donelan (1990) predict enhanced roughness for shoaling conditions.
Verification of the calculated $C_{D10n}$ values

- Simulation of the published datasets

The agreement between the calculated (Eq(6)) and observed $C_{D10n}$ values was good, except the underprediction at higher wind speeds for AD96 (strongly shoaling waves).

The results for the wave-aged-based Eq. (3) show more disagreement compared to that of Eq(6).
Verification of the calculated $C_{D10n}$ values

- Simulation of coastal, lake, and wave tank data

Eq. (6) will be used to predict the $C_{D10n}$ to $U_{10n}$ relationship for various published coastal, lake, and wave tank datasets that were not used in developing the model.

Since the predicted $C_{D10n}$ values vary little with duration or retch, these wave estimates should be adequate for the purpose, provided the waves were not depth limited and significant swell was not present.
Verification of the calculated $C_{D10n}$ values

(1) VM97 data for offshore winds

The majority of the observations (75%) occurred in the range $4 \text{ m/s} < U_{10n} < 9 \text{ m/s}$ where the predicted $C_{D10n}$ and the VM97 data are in reasonable agreement.
Verification of the calculated $C_{D10n}$ values


Very similar $C_{D10n}$ were predicted using Eq. (6) compared to the RASEX offshore wind since both have the similar situation.
Verification of the calculated $C_{D_{10n}}$ values

(3) Wave tank

For both these experiments the waves would have been in deep water.

The $C_{D_{10n}}$ values predicted by Eq. (6) are in both cases being lower than that

The implied $C_{D_{10n}}$ to $U_{10n}$ relationship is about 10% lower than predicted but, above 5m/s, shows a very similar variation with wind speed.

FIG. 8. Further examples of observed and predicted for pure wind seas. The continuous line is the predicted $C_{D_{10n}}$ to $U_{10n}$ relationship for a fully developed pure wind sea. The dotted gray line is the HEMAX relationship from Oost (1998). The gray circles are wave tank data from Cheng and Mitsuyasu (1992), the gray chain line is the wave tank relationship found by Keller et al. (1992), the dashed line shows the values predicted for the wave tank data using Eq. (6).
Verification of the calculated $C_{D10n}$ values

(4) The Donelan (1982) data

For Lake Ontario, Eq. (6) predicts $C_{D10n}$ values similar to or greater than those observed for onshore winds (older wave age or longer fetch), but it does not predict the high $C_{D10n}$ values observed during offshore winds and very short fetch.
Verification of the calculated $C_{D_{10n}}$ values

- Simulation of open ocean data

  The data used in developing Eq. (6) are from lake or coastal sites, so finally open ocean values for $C_{D_{10n}}$ will be considered for the deep ocean.

  The wave information (from SWS-2) represents a mixture of windsea and swell and does not contain directional information.

  The initial point by point comparison of the predicted and observed $C_{D_{10n}}$ was disappointing with poor correlation ($R^2=0.44$).
Verification of the calculated $C_{D10n}$ values

- Simulation of open ocean data

**Fig. 10.** Calculated values of drag coefficient, $C_{D10n}$, vs observed values for the SWS-2 experiment, the data have been averaged in intervals of 1 m s$^{-1}$. Error bars show the standard error of the mean. The regression line shown is $C_{D10n}(calc) = -0.21 + 1.08C_{D10n}(obs)$ ($R^2 = 0.97$).

Mean values were calculated for 1 m/s increments of $U_{10n}$, the agreement was good with $R^2 = 0.97$ (Fig. 10).

**Fig. 11.** Mean observed values of the drag coefficient, $C_{D10n}$ (gray line), and values calculated from Eq. (6) (black line) plotted against the wind speed, $U_{10n}$, for the SWS-2 data. The data have been averaged in 1 m s$^{-1}$ ranges. Error bars show the standard error of the mean. The thick dashed line was calculated using Eq. (6) for a fully developed sea. The chain line is the Yelland et al. (1998) relationship.
Verification of the calculated $C_{D10n}$ values

Because of the dominant long wavelength swell contribution, the values for the slope, $H_s / L_p$, were low compared to a fully developed sea at lower wind speeds.

At wind speeds greater than about 12-15 m/s, the wind sea was dominant over the swell.
Discussion

• Since the effect of the steepness of young waves is cancelled by the small height, Eq. (6) falsely predicts the roughness of waves at short fetch.

• For deep water wind waves, in the absence of swell, for practical purpose, a relationship is defined

\[
10^3C_{D10n} = 0.87 + 0.0752U_{10n} - 0.000661(U_{10n})^2
\]

\[
(5 \leq U_{10n} \leq 30 \text{m/s})
\]

Which well represents a number of datasets.

• Using the observed wave parameters, Eq. (6) successfully predicted \(C_{D10n}\) observations that departed from the expected deep water windsea relationship approximated by Eq. (8).

• The possible explanations for the failure of Eq. (6) to predict the short fetch Lake Ontario data are the difficult-observed young waves and the data bias for some reason.
Discussion

- It is surprising that Eq. (6) correctly predicts the observed $C_{D10n}$ values of open ocean since no open ocean data are used in developing the formula.
Summary

- The function of the wave slope, \( z_o / H_s = A(H_s / L_p)^B \), is proposed to predict the roughness length, which reconciles many anomalies caused by studying relationships between the Charnock parameter and the wave age.

- The formula correctly predicts the behavior of a number of published field datasets, that is, higher \( C_{D10a} \) values is predicted at lower to moderate wind speeds and a less rapid increase of \( C_{D10a} \) with increasing \( U_{10n} \) compared to the open ocean.

- The formula predicts well the magnitude of \( C_{D10n} \) in the open ocean which includes the effect of swell changing the effective wave height and steepness.

- The roughness length formulation advocated here will allow further progress since until now it has been continually hindered by the use of wave age formulation.
Thank you